Low Order Electro-Quasistatic Field Simulations Based on Proper Orthogonal Decomposition

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Abstract — The application of Proper Orthogonal Decomposition to the Galerkin Finite Element formulation of transient electro-quasistatic (TEQS) fields is presented. This approach results in a low order approximation by projection methods of the discrete transient electro-quasistatic system of ODEs. First numerical results for constant and nonlinear conductive materials are shown for TEQS simulations.

I. Introduction

The designs of high-voltage (HV) devices are getting more sophisticated and more and more parameter has to be taken into account. Hence parameter studies and optimization challenging the development of simulation tools in high-voltage engineering. The electro-quasistatic (EQS) field assumptions [1], [2] are suitable for most HV applications, because HV devices are operated at low frequencies. The Galerkin Finite Element Method applied to the transient electro-quasistatic (TEQS) field equation [2], [3] is able to treat slow-varying fields with capacitive and nonlinear resistive effects, as they are important for high-voltage technology. This often leads to large scale ordinary differential equation (ODE) systems. One method to facilitate parameter studies and optimization is to approximate the large scale systems by lower order approximations. In this work, the proper orthogonal decomposition (POD) is applied to the discrete EQS system. The POD method is a commonly used projection method for linear and nonlinear systems [4], [5].

II. FEM discretization

A. Transient Electro-Quasistatic Fields

The transient simulation of HV field problems requires a formulation that takes dielectric effects as well as nonlinear conductive effects into account. Neglecting inductive effects ($\partial_t \vec{\mathbf{B}} = 0$) in Faraday's law and using Poincaré's identity ($\mathrm{curl}\ \mathrm{grad} \equiv 0$) leads to the EQS equation

$$\mathrm{div}\,\varepsilon\,\mathrm{grad}\,\partial_t\varphi + \mathrm{div}\,\kappa(|\mathrm{grad}\,\varphi|)\,\mathrm{grad}\,\varphi = 0 \qquad (1.1)$$

with $\vec{\mathbf{E}} = -\mathrm{grad}\,\varphi$. The permittivity is denoted by ε and the conductivity κ may depend of the electric field strength.

B. Galerkin Finite Element Discretization

The spatial discretization of complex models by Galerkin Finite Element Method yields into a large nonlinear ODE system

$$\mathbf{C}(\phi(t))\,\phi(t) + \mathbf{P}\frac{\mathrm{d}}{\mathrm{dt}}\phi(t) = \mathbf{b}(t)\,, \qquad (1.2)$$

Where $\mathbf{C}$ is the nonlinear conductivity matrix, $\mathbf{P}$ the constant permittivity matrix and $\mathbf{b}$ the right-hand-side vector which incorporates Dirichlet boundary conditions. Due to the large differences in material parameters the system (1.2) is a stiff ODE system. Hence the solution can be realized by implicit time integration with the backward differtiation (BDF1) Euler method [3] or a Singly-Diagonal-Implicit-Runge-Kutta (SDIRK) method [2]. Therefore in each time step one or several high dimensional nonlinear system of equations must be solved. In this work only the BDF1 integration is used.

III. Proper Orthogonal Decomposition

The proper orthogonal decomposition is a method for building a low-order approximation of both linear and nonlinear dynamical systems. The method is based on the formulation of the system behavior by extracting a low number of orthogonal vectors by the observation of the system dynamics. The number of p observations of the n dimensional system vectors are assembled in a so-called snapshot matrix

$$\mathbf{X} = \left[\phi(t_1),\ldots,\phi(t_p)\right] \in \mathbb{R}^{n\times p}\ . \qquad (1.3)$$

The POD approximates the high dimensional observations matrix by a lower dimensional matrix which is usually built up from a small number of orthogonal column vectors.

A. POD and Singular Value Decomposition

These orthogonal vectors can be computed by the singular value decomposition (SVD). Every matrix $\mathbf{X}$ can be decomposed into three matrices by the SVD and it can be expanded into a rank-one decomposition

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sigma_1\mathbf{u}_1\mathbf{v}_1^T + \cdots + \sigma_r\mathbf{u}_r\mathbf{v}_r^T\,. \qquad (1.4)$$

The columns of $\mathbf{U}$ and the rows of $\mathbf{V}$ contain the orthonormal left and right singular vectors $\mathbf{u}_i$ and $\mathbf{v}_i$. The diagonal matrix $\mathbf{\Sigma}$ features the positive singular values σ_i ordered from large to small on the diagonal. The higher order terms in the rank-one decomposition become small rapidly with increasing index *i*. A low-order approximation of the snapshot matrix can be achieved by dropping out the small singular values.

$$\mathbf{X} \approx \mathbf{X}_k = \mathbf{U}_k\mathbf{\Sigma}_k\mathbf{V}_k^T = \sigma_1\mathbf{u}_1\mathbf{v}_1^T + \cdots + \sigma_k\mathbf{u}_k\mathbf{v}_k^T\,, \qquad (1.5)$$

with $k \ll r = \mathrm{rank}(\mathbf{X})$. The matrix $\mathbf{U}_k$ is in $\mathbb{R}^{n\times k}$. This matrix can be used as a projector to map from space of dimension k to the space of the full system with dimension *n*.

B. POD and TEQS

Let $\phi(t)=\mathbf{U}_k\mathbf{x}(t), \quad \mathbf{x}(t)\in\mathbb{R}^k$. Then system (1.2) can be reduced to the reduced-order system

$$\mathbf{U}_k^T\mathbf{C}(\mathbf{x}(t))\mathbf{U}_k\ \mathbf{x}(t)+\mathbf{U}_k^T\mathbf{P}\ \mathbf{U}_k\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{x}(t)=\mathbf{U}_k^T\mathbf{b} \qquad (1.6)$$

The matrices $\mathbf{U}_k^T\mathbf{C}(\phi(t))\mathbf{U}_k$ and $\mathbf{U}_k^T\mathbf{P}\ \mathbf{U}_k$ are $k\times k$ and the vector $\mathbf{U}_k^T\mathbf{b}$ is of dimension k. The implicit time integration and the solution of the nonlinear equation systems can now be performed for the low dimensional system.

IV. First Numerical Results

The numerical experiments are performed with a 2d axis-symmetric TEQS-code implemented in MATLAB. As a first numerical benchmark, a surge arrester model discretized with 1280 triangles, 719 Nodes (668 Nodes without boundary nodes) was excited with a ramped sinusoidal signal of 50Hz.

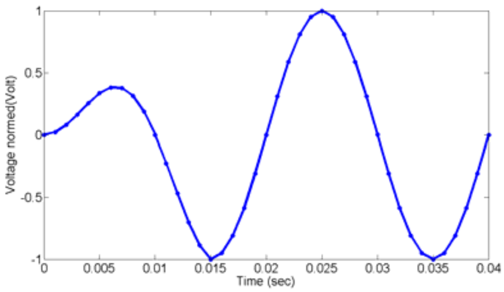

Fig. 1 Ramped sinus for two periods

For first numerical experiments the snapshot-matrix was assembled by 40 time steps of a full simulation. Aiming to save computing time, only the elements which contain nonlinear materials are (re-)assembled in every time step.

A. Constant conductivity

For constant permittivity and constant conductivity the singular values become small very fast. As may be assumed the dominant mode in the projection matrix is the static field solution, i.e., $\mathbf{u}_1$, the first column vector of $\mathbf{U}_k$ represents the electrostatic mode up to a scaling factor.

TABLE I
REDUCED ORDER FOR NONLINEAR MATERIALS

Surge arrester	Constant κ	Nonlinear κ
Dim full order system	668	668
σ_1	50.45	$50\cdot10^6$
σ_8	$1.69\cdot10^{-10}$	$1.1\cdot10^{-3}$
σ_{16}	$>10^{-15}$	$>10^{-5}$
Dim reduced system	8	16
Reduction factor	83.5	41.8
Maximal deviation in nodal potential $\max\limits_{\forall\text{ nodes}}\lVert\phi_{\text{full}}-\phi_{reduced}\rVert$	$2.5\cdot10^{-12}$ kV	0.3 kV
$\max\limits_{\text{nodes}}\lVert\phi_{\text{full}}\rVert$	471 kV	471 kV
speedup factor	6.2	1,4

B. Nonlinear conductivity

The low speedup factor results from the singular value decomposition and the bad conditioned low order equations system which has to be solved in the Newton solver.

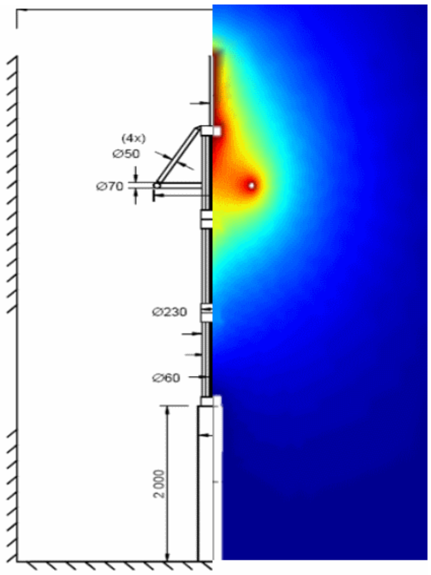

Fig. 2 IEC 60099-4 surge arrester in static mode

The results in Table I show that for both constant and nonlinear material behavior the TEQS system dimension reduction factor of about two orders of magnitude is achieved. For a speed-up of the CPU time further implementation and formulation improvements can be expected.

V. Conclusion

A Proper Orthogonal Decomposition model order reduction scheme for transient electro-quasistatic field simulations was proposed. The method applies to formulations with constant and/or field dependent nonlinear material conductivities. Numerical results showed a considerable reduction of the system dimensions and a possible speed-up of the simulation times. The upcoming full paper will feature additional details on the model order reduction formulation and additional numerical test results.

VI. References

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